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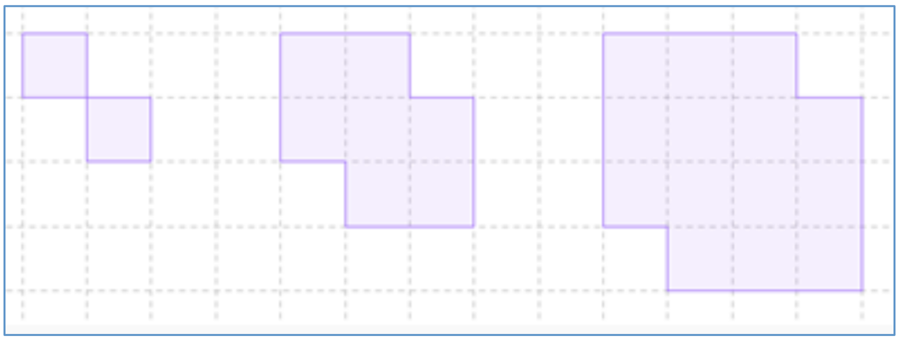
Linear and Exponential Functions Study Guide

Generalizing Patterns

We can write function rules based on patterns by analyzing the patterns we are given.
The first step in analyzing these patterns and writing a rule is to create a table of values.



Step	Squares
1	2
2	7
3	14

Questions to consider:

1. Is the pattern growing by the same amount with each new step (i.e. is the pattern LINEAR?)

2. Is the pattern being multiplied by the same amount with each new step (i.e. is the pattern EXPONENTIAL?)

3. If your answer is NO to the questions above, the pattern is likely some type of squaring pattern. At this point, I would look closely at the images to discern what is happening, and might help you convert this to a mathematical rule.

Example:

In the example above, there is no adding or multiplying pattern from step 1 to 2 to 3, so I took a look at the picture. I see in step 1, there is a 2 x 2 square with the top-right and bottom-left squares missing. I see that in step 2, there is a 3 x 3 square with the top-right and bottom-left square missing, and I see the pattern repeat itself in the third step of the pattern. I see that the side length of each larger square is one more than the step number, and we are always subtracting 2…hence I get a rule of $\boldsymbol{f(n) = (n+1)^2 - 2}$, where n is the step number and $f(n)$ is the number of squares.

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PROPERTIES OF FUNCTIONS

A **function** is a correspondence between two sets X and Y in which each element of X is matched to one and only one element of Y.

Example:
The assignment of people to social security numbers. Each person corresponds to one and only one social security number.

Non-example:
The correspondence of students to their pets. (this is because students may have more than one pet).

Domain: the domain of a function is the set X of all possible inputs.

Range: the range of a function is the subset of Y denoted by $f(x)$ and is defined by the following property: y is an element of $f(x)$ if and only if there is an x where $y = f(x)$.

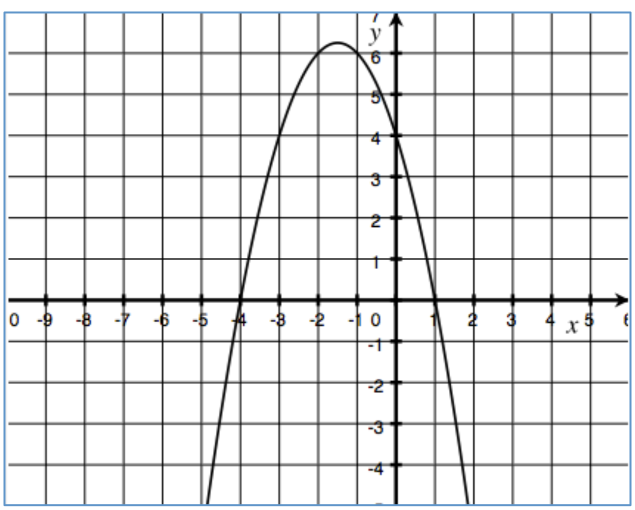
Functions are **increasing** on an interval I if $f(x_1) < f(x_2)$whenever $x_1 < x_2$ in I.

Function are **decreasing** on an interval I if $f(x_1) > f(x_2)$whenever $x_1 < x_2$ in I.

Functions are **positive** on an interval I if $f(x) > 0$ for all x in I.

Functions are **negative** on an interval I if $f(x) < 0$ for all x in I.

Example:



The function is **increasing** on the interval $(-\infty, -1.5)$

The function is **decreasing** on the interval $(-1.5, \infty)$

The function is **positive** on the interval $(-4, 1)$

The function is **negative** on the intervals $(-\infty, -4)$, $(1, \infty)$

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LINEAR FUNCTIONS

Linear functions grow additively. We have studied linear functions all year long. They are written in the form:

$$\boldsymbol{f(x) = mx + b}$$

Things to know:

- $f(0) = b$. In words, this means when $x = 0$, the b value is equal to $f(x)$. It is the y-intercept when the function is graphed.
- m defines the additive growth of the function over every interval length of 1. In English, for every one unit of change in x, the function $f(x)$ changes by m units.
- The point at which $f(x) = 0$ is the x-intercept.

EXPONENTIAL FUNCTIONS

Exponential functions grow multiplicatively. They are written in the form:

$$\boldsymbol{f(x) = a(b)^x}$$

Things to know:

- $f(0) = a$. In English, this means that when $x = 0$, the function $f(x)$ is equal to the value of a.
- a is the 'starting' point of the function. It is the y-intercept when the function is graphed. .
- b defines the growth (or decay) factor of the function. When $b > 1$, the function will grow exponential large. When $0 < b < 1$, the function will decay (shrink).

EXPONENTIAL GROWTH FUNCTIONS WILL ALWAYS ULTIMATELY GROW FASTER THAN LINEAR FUNCTIONS!!

Example:

Linear Larry begins with $10,000 in his savings account. He earns no interest, but deposits $100 every year.

Exponential Ellie begins with $100 in her savings account, but earns annual interest of 10%.

It might take 53 years, but eventually, that $100 will be worth more than the $10,000.

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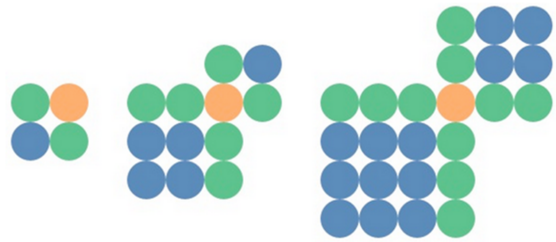
PROBLEM SET

Hand this in before May 29!! This may not be turned in late…we've got no extra time!

1. Study the pattern below, and answer the questions.



a. Write a function rule that describes the pattern. What do you see as the pattern?

b. How many dots will there be on the 12th pattern?

2. Study the table below that captures the balance of Molly's bank account at the end of each month and answer the questions.

Month	Dollar
0	$250
1	$500
2	$1,000
3	$2,000

a. Write a function rule that describes the pattern.

b. If the pattern continues, how many months will it take Molly to be a millionaire?

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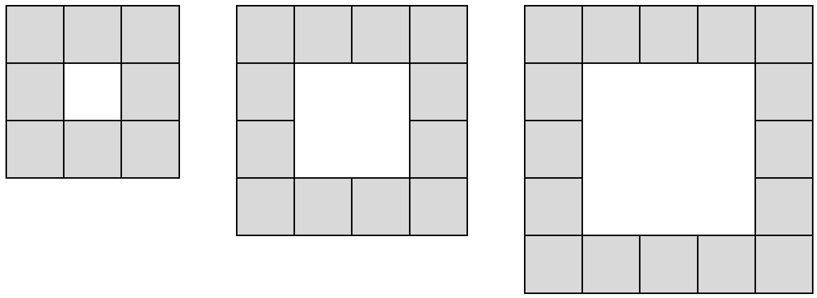
3. Study the pattern below and answer the questions.



a. Write a sentence or two description of what you see happening in this pattern to the number of shaded squares.

b. Write a function rule that describes the pattern in the shaded squares.

c. How many shaded squares will there be in pattern 43?

4. Emma is shopping for fruit. She buys one red delicious apple, a carton of strawberries, a box of blueberries, one orange, a banana, and a lemon.

a. Is the assignment of fruit to colors a function? Explain your answer. If so, what is the domain and what is the range?

b. What does $f(lemon)$ equal?

c. Is the assignment of colors to fruit a function? Explain your answer. If so, what is the domain and the range.

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5. Julia's theatre company is putting on a production of Bye Bye Birdie, and there are 14 performers for 18 roles. This means that some performers will have to play more than one role. Is the assignment of roles to performers considered a function? Explain your answer.

6a. Is the assignment of state capitals to states in the US a function? Explain your answer.

6b. What does $f(Montpelier) = Vermont$ mean?

6c. What does $f(Los\ Angeles)$ mean?

7. A Major League Baseball typically has a 25-man roster—meaning 25 baseball players are available to play each day. During each game, only 9 players can be on the field at the same time—some players share positions.

a. Is the assignment of positions to players considered a function. Why or why not?

b. What would $f(pitcher)$ mean in the context of the situation?

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8. An exponential function is defined as $f(x) = 5(3)^x$.
a. Fill in the table below.

x	$f(x)$
0	
1	
2	
3	
4	

b. Describe in words what is happening to the function.

9. An exponential function is defined as $g(x) = 3(0.5)^x$
a. Fill in the table below.

x	$g(x)$
0	
1	
2	
3	
4	

b. Why is your output decreasing as *x* grows larger?

10. Write 3 different polynomial functions such that $f(6) = 19$

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11. Catherine is thinking about her future, and she decides to invest $5,000 in a certificate of deposit (CD) that gives her 4% interest each year.

a. How much money will she have in her CD at the end of the first year?

b. How much money will she have in her CD at the end of 5 years?

c. What function rule can we apply to help determine the value of CD for any number of years?

12. A biology lab is studying the growth of a virus at certain times. A researchers records data every day noting the number of cells observed at 9AM every day. Below is a table of his observations.

Day	0	1	2	3	4	5	6
Number of cells observed (in thousands)	3	6	12	24	[redacted]	[redacted]	192

a. Unfortunately, ink spilled on the observation sheet. If the pattern should hold, how many cells were observed on day 4 and 5?

b. Write a function rule for how this virus grows over time.

c. Is this a linear function, an exponential function, or neither? Explain how you know.

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13. Earlier in the year, we studied lines of best fit, and looked at several different real world scenarios and created a linear equation that best represented the data. An example below looks at Detroit's population over the past 60 years.

Year	Population
1950	1,849,568
1960	1,673,144
1970	1,511,482
1980	1,203,339
1990	1,027,974
2000	951,260
2010	713,144

When we calculated the line of best fit, we came up with a linear function, $f(x) = -19200x + 1{,}849{,}568$, where x was the number of years that have passed since 1950.

a. Based on this data, in what year would Detroit have a population of 0? Do you think that Detroit will ever have a population of 0?

b. We can also use an exponential function in the form $f(x) = ab^x$. If we say on average that Detroit expects to lose about 1% of its population each year, can you write the exponential function that would describe Detroit's growth.

c. WITHOUT doing calculations, do you think that using a linear model or exponential model is better in this instance? Or do you think neither is better? Explain your answer.