NAME:_____ Math _____, Period _____

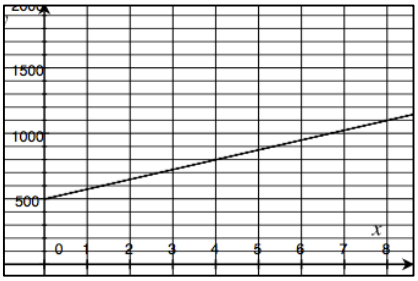
Mr. Rogove Date:_____

Linear Functions And Geometry
Study Guide

FUNCTIONS

Functions are rules that assign each input exactly one output.

We have described functions in four different ways:

<table>
<tr>
<td><u>Verbally/Written Description</u>
I have $500 in my bank account now, and deposit $75 per week.</td>
<td><u>Equation</u>
$y = 75x + 500$ or
$f(x) = 75x + 500$</td>
</tr>
<tr>
<td><u>Table</u>
<table>
<tr><th>Weeks (x)</th><th>Money (y)</th></tr>
<tr><td>0</td><td>500</td></tr>
<tr><td>1</td><td>575</td></tr>
<tr><td>2</td><td>650</td></tr>
<tr><td>3</td><td>725</td></tr>
<tr><td>5</td><td>875</td></tr>
</table>
</td>
<td><u>Graph</u>
 

</td>
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</table>

A **linear function** is a special kind of function where the function rule is specifically a linear equation in the form $y = mx + b$.
Characteristics of Linear Functions:

- The rate of change of a linear function stays constant.
- When the slope of a linear function is negative, the function is decreasing.

  When the slope of linear function is positive, the function is increasing.
- Linear functions graph as straight lines.
- Linear functions describe proportional relationships.

<u>REMINDER</u>: Some functions don't involve numbers at all.
<u>Example</u>: Input is car model (i.e. Accord), and output is car manufacturer (i.e Honda).

When we **<u>talk</u>** about functions, we say that the output is a function of the input.
<u>Example</u>: The money in my bank account is a function of the number of weeks I've saved.

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WHAT DOES A FUNCTION LOOK LIKE?

**In a table of values, there are NO *x*-values (input values) repeated

**On a graph, it means that a vertical line will only pass through the function ONCE.

Discrete v. Continuous Functions

A **discrete function** is a function that only has a specific set of inputs (such as integers).

Example: A box of cookies costs $3.00. You can't buy a fractional box of cookies.

A **continuous function** is a function that could include rational number input values.

Example: A pound of grapes is $3.00. You can buy 3.5 pounds of grapes.

GEOMETRY (Volume of 3D shapes)

Remember these formulas

Cylinder

$$V = (\pi r^2)h$$

$V = (area\ of\ base) \times height$

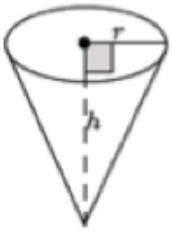
Height

Cone

$$V = \frac{1}{3}(\pi r^2)h$$

$V = \frac{1}{3}(area\ of\ base) \times height$

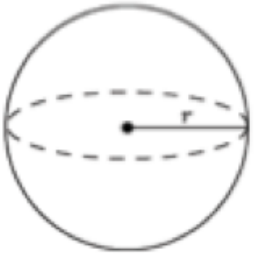
The base is still a circle!



The vertex is the pointed part of the cone

Sphere

$$V = \frac{4}{3}\pi r^3$$



NAME:_____ Math _____, Period _____

Mr. Rogove Date:_____

PROBLEM SET

Please complete all problems and submit as you take this assessment

1. Rachel is hiring a plumber to re-pipe her kitchen and bathroom. One plumbing company, LeakProof Plumbing Company, is charging a $500 for materials plus $120 per hour. Another company, DripFreeSince2003 Inc., does not charge for material, but their hourly rate is $165. A third company, CleanYerPipes.com, submits a bid to do the work for $2300 no matter how long it takes. No company provides an estimate of how many hours it will take.

a. Write linear equations that model the charges for each of the three companies.
Leakproof: $y = 500 + 120x$
DripFreeSince2003: $y = 165x$
CleanYerPipes.com: $y = 2300$

b. If it takes 8 hours for the work to be completed, which company will provide the best value?

Leakproof: $y = 500 + 120(8) = 500 + 960 = \$1460$
DripFreeSince 2003: $y = 165\ (8) = \$1320$
CleanYerPipes.com: $2300

Drip Free is the cheapest…

c. For what time interval is LeakProof Plumbing Company the cheapest alternative?
DripFree is cheapest from 0 hours to approximately 11 hours and 7 minutes.
LeakProof is cheapest if your job will take longer than 11 hours, 7 minutes, but less than 15 hours.

d. At what point does it become most economical to hire CleanYerPipes.com?
CleanYerPipes.com is cheapest if the job will take longer than 15 hours.

NAME:_____ Math _____, Period _____

Mr. Rogove Date:_____

2. LIGHT BULBS. Incandescent light bulbs have been used for many decades to provide light in homes and businesses. They cost about $1.50 each, and they last about 1,000 hours. Newer CFL light bulbs have been introduced that are more expensive—costing $10.00 each, but they last about 8,000 hours.

a. After how many hours will it pay to get the CFL bulbs? (think about using linear functions to help you answer).
If you use the incandescent bulbs for 6,000 hours, you'll spend $9.00 (because you'll need 6 bulbs at $1.50 each). If you use the incandescent bulbs for 6,001 hours, you'll need a 7th bulb and the total cost for the bulbs will rise to $10.50, making it more expensive than the $10.00 CFL bulb.

b. To make things more interesting, what if I told you that the incandescent bulbs are much more inefficient, and they add $.01125 per hour to your electric bill. CFL bulbs will only add $0.00345 to your electric bill for each hour they are in use. How will this affect your determination of when it's more economical to buy incandescent bulbs v. CFL bulbs? Explain your answer using words and equations.
Now, we can use equations…
The equation for the incandescent bulb is as follows:
$C = 0.01125h + 1.50$ where C is the cost in dollars, and h is the number of hours the bulbs are in use (note, that we need to add another $1.50 for every 1000 hours used—this will graph like a piece wise function.)

The equation if the CFL bulb:
$C = 0.00345h + 10.00$ again where C is the cost in dollars and h is the number of hours the bulbs are in use. (note we need to add another $10.000 for every 8000 hours used—this will also graph as a piece wise function.

If we set the above equations equal to one another we find that at about 1,090 hours, the cost would be equal, but we would have had to buy a second incandescent bulb, so let's assume we buy 2 incandescent bulbs, making the first equation: $C = 0.01125h + 3.00$. Using this number, we would find that the breakeven point would be about 897 hours…so, armed with this information, we can figure out that after 1,001 hours, you'd have to buy another incandescent bulb and at THAT point, you'd need a new bulb and that would make the incandescent bulbs more expensive.

Another way to look at it as follows: if you used each for 1000 hours: Incandescent would cost $0.01125 \times 1000 + 1.50 = \$12.75$. the CFL would be $0.00345 \times 1000 + 10.00 = \$13.40$. At that point, you'd have to spend another $1.50 on the incandescent bulb.

c. Even more expensive than CFL bulbs are newer LED light bulbs. LED light bulbs cost about $20 each but they last for an amazing 25,000 hours, and they cost almost nothing to use per hour (the cost is $0.00135) After how many hours of use will the LED bulbs be the most economical? Justify your answer.

We can use the same logic to see when the CFL bulbs become more expensive….but the short answer (without using any equations) is that as soon as you buy the second CFL bulb (after 8,000 hours) you'll be spending more simply because at that point, the cost of the bulbs will be equal, and the cost of keeping the lights on is much cheaper for the LED bulbs…so after 8001 hours, the CFL bulbs will be more expensive.

d. If the typical household has their lights on 4 hours a day, how many YEARS would you expect each type of bulb to last?

Incandescent: 4 hours a day for 1000 hours means the bulb would last 250 days.

$$\frac{250}{365} = 0.685\ years$$

CFL: 4 hours a day for 8000 hours mean the bulb would last 2000 days.

$$\frac{2000}{365} = 5.48\ years$$

LED: 4 hours a day for 25,000 hours means the bulbs would last 6250 days.

$$\frac{6250}{365} = 17.12\ years$$

Based on this usage, if your parents installed an LED bulb when you were born, you'd be able to use them to shine light on your college applications.

3. POWERBALL. The recent powerball lottery was worth $1.6Billion. Winners can decide to take a lump sum payout or they can get 30 annual payments. If they decide to take a lump sum, winners could expect approximately $992,000,000. Of course, the winners wouldn't get ALL that money. There are taxes to pay-the federal government will shave 39.6% from the total.

a. How much is the lump sum payment AFTER taxes are taken out?

This would 60.4% of $992,000,000 or $599,168,000.

b. How much is each annual installment AFTER taxes are taken out?

This would be 60.4% of 1,600,000,000 divided by 30.

$$\frac{0.604(1{,}600{,}000{,}000)}{30} = \$32{,}213{,}333.33$$

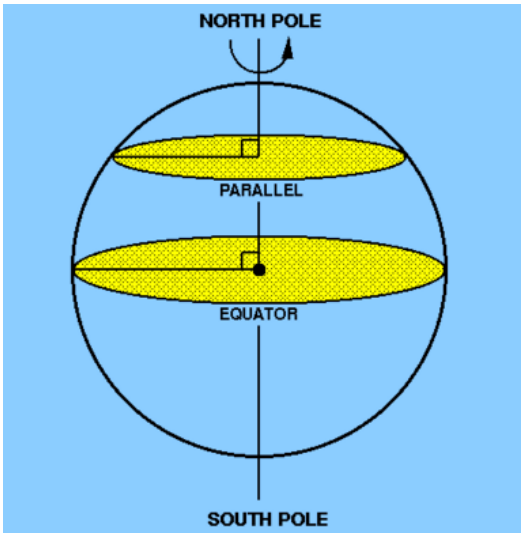
c. If you take the lump sum payment, how much money (in dollars) do you have to earn each year for the lump sum to turn out to be a better investment than the annual installments? Explain how you figured this out.

If you take the lump sum, you'll end up with $599,168,000. If you take the installments, you'll eventually get $966,400,000. This means that over the course of 30 years, you'll have to invest your money in something that will get you an average of $\frac{966{,}400{,}000-599{,}168{,}000}{30}$ in order for the lump sum to be more lucrative. This equates to $12,241,066.67 each year for 30 years that you need to make in order for the lump sum to be worth as much money as the annual installments for 30 years. This $12Million Plus equates to a return on your investment of a little more than 2% each year. This doesn't take into account the concept of compounding interest. Historically, it's easy to make a 2% return on your investment and this is why many people take the lump sum.

d. What would you do? Explain why.

I'd definitely take the lump sum. See above for rationale.

4. Assume the earth is perfectly round and that the equator is a good measure of the absolute largest circumference. If the equator is 24,900, what is the volume of the earth? (express your answer in terms of pi).



$V_{sphere} = \frac{4}{3}\pi r^3$ The circumference of 24,900 can help us find the radius…because $C = 2\pi r$…so let's solve for r.

$$24900 = 2\pi r$$

$$\frac{12450}{\pi} = r$$

So, armed with this info, we can find the volume in terms of pi…

$$V = \frac{4}{3}\pi\left(\frac{12450}{\pi}\right)^3$$

$$= \frac{\frac{4}{3}(1929781125000)}{\pi^2}$$

$$= \frac{2573041500000}{\pi^2}\ cubic\ miles$$

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Mr. Rogove Date:_____

5a. Many ice cream cones are 1.5 inches in diameter at the top, and would stand about 4 inches tall. How much ice cream would be able to fit inside the cone (assume that ice cream does not pile on top of the cone, but is leveled at the top of the cone).

$$V_{cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi \left(\frac{3}{4}\right)^2 (4)$$

$$= \frac{3}{4}\pi \; cubic \; inches$$

5b. Using the information from above, let's say instead of cones, they made "ice cream cylinders" for you to carry your delicious iced dairy treat. What are possible dimensions for a cylinder that will **twice** as much ice cream as the cone above?

The cylinder would need to have a volume of 1.5π cubic inches. If the diameter of the cylindrical cone were 2 inches, and the radius were 1 inch, the height would be 1.5 inches in order to get twice as much ice cream.