NAME:___Key_____   Math 7.2, Period _____

Mr. Rogove   Date:_____

STUDY GUIDE: SOLVING LINEAR EQUATIONS

Writing Equations

When reading word problems and trying to create an equation using symbols, it is extremely important to begin by defining your variable.

Example: Your family is taking a road trip to Chicago. They drive 450 miles on the first day, $\frac{1}{3}$ of the entire distance on the second day, $\frac{1}{4}$ on the third day, and finally drive the final 325 miles on the fourth day.
Let d be the distance of the entire trip in miles.

Try to break more complicated word sentences down into more manageable pieces.

Example: When a number is taken away from 57, what remains is four more than 5 times the number.
Let x be the number.
"When a number is taken away from 57" → $57 - x$
"What remains is" → $=$
"Four more than 5 times the number → $5x + 4$

Solving Linear Equations

Linear Expressions contain constants, variables raised to the first power, or variables (raised to the first power) multiplied by coefficients.

Examples:	_Non-Examples:_
12	$3x^3$
$-3x$	$\frac{1}{4x^8} + 19$
$\frac{1}{4}x - 2$	$x^2 - 1$
$32 + 21x$	$(x + 3)(x - 2)$

> WHEN SOLVING LINEAR EQUATIONS,
> YOUR ULTIMATE GOAL IS TO **ISOLATE THE VARIABLE,**
> ALSO CALLED **SOLVING FOR X.**

CHECKING YOUR SOLUTION IS A PART OF SOLVING THE LINEAR EQUATION!!

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In order to keep your equation in balance, use the following properties of equality

Addition Property of Equality	Subtraction Property of Equality
If $A = B$, then $A + C = B + C$	If $A = B$, then $A - C = B - C$
Multiplication Property of Equality	**Division Property of Equality**
If $A = B$, then $A \times C = B \times C$	If $A = B$ $(and\ C \neq 0)$, then $\dfrac{A}{C} = \dfrac{B}{C}$

Different Moves to Make When Solving Equations

When solving equations you might have to do one or more of the following moves:
Combine Like Terms before your use properties of equality.
Example: $3x + 4 - 5x + 13 = 26$ → $-2x + 17 = 26$

Gather all variable terms on one side of the equation before using properties of equality to isolate your variable.
Example: $3x - 23 + 4x = 24 + \frac{1}{2}x - 21$ → $7x - \frac{1}{2}x - 23 = 24 - 21$

Use the distributive property to clear parentheses prior to using properties of equality.
Example: $43x - 3(21 + 12x) = 3 - (4x - 5)$ → $43x - 63 - 36x = 3 - 4x + 5$

Use Rules of Proportions to create linear equations when your equations contain a pair of fractions.
Example: $\dfrac{3x-4}{13x+\frac{1}{3}} = \dfrac{5}{7}$ → $7(3x - 4) = 5\left(13x + \frac{1}{3}\right)$

Classifying Solutions to Linear Equations

One Solution	No Solution	Infinitely Many Solutions
Example: $\quad 7x - 3 = 5x + 5$	Example: $\quad 7x - 3 = 7x + 5$	Example: $\quad 7x - 3 = -3 + 7x$
-Different coefficients -Same or different constant terms (If the same constant term, then $x = 0$)	-Same coefficients -Different constant terms	-Same coefficients -Same constant terms

NAME: KEY   Math 7.2, Period _____

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The Problem Set

Complete this problem set before you take the test! If you are stuck or have specific questions about any of these problems, come find me before the test so we can address your concerns.

Write an equation for each word problem and answer the question.

Shannon's goal is to run 40 miles each week. This week, she has already run distances of 5.3 miles, 6.5 miles, and 6.2 miles. If she wants to spread out the remaining miles evenly over the next four days, how many miles per day does she need to run each day? Write an equation and solve.	The Y charges a $44 sign up fee and $30/month for an individual gym membership. CitySports Fitness charges a $99 sign up fee and charges $25/month for membership. Write an equation that will help you determine how many months it will take for the total cost of membership to be the same? How many months will it take?

Let x = the # of miles Shannon runs the last 4 days.

$$5.3 + 6.5 + 6.2 + 4x = 40$$
$$18 + 4x = 40$$
$$-18 \qquad -18$$
$$\frac{4x}{4} = \frac{22}{4}$$
$$x = 5.5$$

Shannon runs 5.5 miles/day

Let x = # of months at the gym.

$$30x + 44 = 25x + 99$$
$$-25x \qquad -25x$$
$$5x + 44 = 99$$
$$-44 \quad -44$$
$$\frac{5x}{5} = \frac{55}{5}$$
$$x = 11.$$

It will take 11 months for the total cost of membership to be the same.

Two vertical angles are described as follows: one of the angles is four times the quantity of a number added to three, and the other angle is 6 less than thirteen times that same number. What are the measures of both angles?	One acute angle of a right triangle is nine less than two times the measure of the other acute angle. The third angle is the right angle. Draw the triangle and find the angle measures.

$4(x+3)$
$13x-6$

$$4(2+3) = 4 \cdot 5 = 20$$
$$13(2) - 6 = 26 - 6 = 20$$

$$4(x+3) = 13x - 6$$
$$4x + 12 = 13x - 6$$
$$-4x \qquad -4x$$
$$12 = 9x - 6$$
$$+6 \qquad +6$$
$$18 = 9x$$
$$x = 2$$

Each angle is 20°

$2x-9$
x
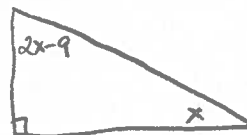
$$x + 2x - 9 = 90$$
$$3x - 9 = 90$$
$$+9 \quad +9$$
$$\frac{3x}{3} = \frac{99}{3}$$
$$x = 33$$

One angle is 33°
The other angle is $2(33) - 9 = 57°$

3

Solve each equation and check your work.

$$-3(4x - 8) - 1 = x + 3(x + 9)$$

$$-12x + 24 - 1 = x + 3x + 27$$

$$-12x + 23 = 4x + 27$$

$$+12x \qquad +12x$$

$$23 = 16x + 27$$

$$-27 \qquad -27$$

$$-4 = 16x$$

$$\boxed{x = -\frac{1}{4}}$$

CHECK:

$$-3(4(-\tfrac{1}{4}) - 8) - 1 = -\tfrac{1}{4} + 3((-\tfrac{1}{4}) + 9)$$

$$-3(-1 \cdot 8) - 1 = -\tfrac{1}{4} + \tfrac{3}{4} + 27$$

$$-3(-9) - 1 = -1 + 27$$

$$27 - 1 \overset{\checkmark}{=} 26$$

$$6(x - 11) - 3(11 - x) = 11(x + 7)$$

$$6x - 66 - 33 + 3x = 11x + 77$$

$$9x - 99 = 11x + 77$$

$$-9x \qquad -9x$$

$$-99 = 2x + 77$$

$$-77 \qquad -77$$

$$-176 = 2x$$

$$\boxed{-88 = x}$$

CHECK!

$$6(-88 - 11) - 3(11 - (-88)) = 11(-88 + 7)$$

$$6(-99) - 3(99) \overset{?}{=} 11(-81)$$

$$-594 - 297 = -891$$

$$-891 \overset{\checkmark}{=} -891$$

$$2\tfrac{2}{3}x + 2 - x = 9(x + 1) + \tfrac{5}{3}x$$

$$\tfrac{8}{3}x + 2 - \tfrac{3}{3}x = 9x + 9 + \tfrac{5}{3}x$$

$$\tfrac{5}{3}x + 2 = \tfrac{32}{3}x + 9$$

$$-\tfrac{5}{3}x \qquad -\tfrac{5}{3}x$$

$$2 = 9x + 9$$

$$-9 \qquad -9$$

$$\frac{-7}{9} = \frac{9x}{9}$$

$$\boxed{x = -\tfrac{7}{9}}$$

CHECK:

$$2\tfrac{2}{3}(-\tfrac{7}{9}) + 2 - (-\tfrac{7}{9}) = 9(-\tfrac{7}{9} + 1) + \tfrac{5}{3}(-\tfrac{7}{9})$$

$$\tfrac{8}{3}(-\tfrac{7}{9}) + 2 + \tfrac{7}{9} = 9(\tfrac{2}{9}) - \tfrac{35}{27}$$

$$-\tfrac{56}{27} + \tfrac{54}{27} + \tfrac{21}{27} = \tfrac{54}{27} - \tfrac{35}{27}$$

$$\tfrac{19}{27} \overset{\checkmark}{=} \tfrac{19}{27}$$

$$2x - 3 - 9x = 14 + x - 1$$

$$-7x - 3 = x + 13$$

$$+7x \qquad +7x$$

$$-3 = 8x + 13$$

$$-13 \qquad -13$$

$$\frac{-16}{8} = \frac{8x}{8}$$

$$\boxed{x = -2}$$

CHECK

$$2(-2) - 3 - 9(-2) = 14 + (-2) - 1$$

$$-4 - 3 + 18 = 14 - 2 - 1$$

$$-7 + 18 = 12 - 1$$

$$11 \overset{\checkmark}{=} 11$$

YAY!

Solve each equation. No need to check your work.

Problem 1:

$$-(-3+13x)-5x = 6\left(\frac{2}{3}x - \frac{2}{5}\right)$$

$$3 - 13x - 5x = \frac{12}{3}x - \frac{12}{5}$$

$$3 - 18x = 4x - \frac{12}{5}$$
$$+18x \quad\quad +18x$$

$$3 = 22x - \frac{12}{5}$$
$$+\frac{12}{5} \quad\quad\quad +\frac{12}{5}$$

$$\frac{\frac{27}{5}}{22} = \frac{22x}{22}$$

$$\boxed{x = \frac{27}{110}}$$

Problem 2:

$$\frac{1}{6}(5x - 12) + \frac{2}{3}(3 - 5x) = -4 - (x + 12)$$

$$\frac{5}{6}x - 2 + 2 - \frac{10}{3}x = -4 - x - 12$$

$$\frac{5}{6}x - \frac{20}{6}x = -x - 16$$

$$-\frac{15}{6}x = -x - 16$$
$$+x \quad\quad +x$$

$$\frac{-\frac{9}{6}x}{-16} = \frac{-16}{-16}$$

$$\boxed{x = \frac{9}{96} = \frac{3}{32}}$$

Problem 3:

$$-5 - (4 - 3x) + 7x = \frac{5}{8}(16x - 20) + 4x$$

$$-5 - 4 + 3x + 7x = 10x - \frac{100}{8} + 4x$$

$$-9 + 10x = 14x - \frac{100}{8}$$
$$-10x \quad\quad -10x$$

$$-\frac{18}{2} = 4x - \frac{25}{2}$$
$$+\frac{25}{2} \quad\quad +\frac{25}{2}$$

$$\frac{\frac{7}{2}}{4} = \frac{4x}{4}$$

$$\boxed{x = \frac{7}{8}}$$

Problem 4:

$$12\left[\frac{2}{3}\left(12 - \frac{9}{2}x\right) = \frac{3}{4}\left(\frac{9}{2} - \frac{3}{2}x\right)\right]$$

$$8\left(12 - \frac{9}{2}x\right) = 9\left(\frac{9}{2} - \frac{3}{2}x\right)$$

$$2\left[96 - 36x = \frac{81}{2} - \frac{27}{2}x\right]$$

$$192 - 72x = 81 - 27x$$
$$+72x \quad\quad +72x$$

$$192 = 81 + 45x$$
$$-81 \quad -81$$

$$\frac{111}{45} = \frac{45x}{45} \quad \boxed{x = \frac{111}{45} = \frac{37}{15}}$$

Solve each equation. No need to check your work.

$$\frac{2x - \frac{3}{4}}{6} \cancel{=} \frac{3 - 2x}{10}$$

$$10\left(2x - \tfrac{3}{4}\right) = 6\left(3 - 2x\right)$$

$$20x - \tfrac{30}{4} = 18 - 12x$$

$$+12x \qquad\qquad +12x$$

$$32x - \tfrac{15}{2} = \tfrac{36}{2}$$

$$+\tfrac{15}{2} \quad +\tfrac{15}{2}$$

$$\frac{32x}{32} = \frac{\frac{51}{2}}{32}$$

$$\boxed{x = \tfrac{51}{64}}$$

$$\frac{5x - 3}{3 - 2x} \cancel{=} \frac{6}{11}$$

$$11\left(5x - 3\right) = 6\left(3 - 2x\right)$$

$$55x - 33 = 18 - 12x$$

$$+12x \qquad\qquad +12x$$

$$67x - 33 = 18$$

$$+33 \quad +33$$

$$\frac{67x}{67} = \frac{51}{67}$$

$$\boxed{x = \tfrac{51}{67}}$$

Please classify each solution as having one unique solution, no solution, or infinitely many solutions. You should simplify the expressions, but there is no need to solve the equations.

$$(9x + 12 - 3x) \div 3 = 2(2 + x)$$

$$\frac{6x + 12}{3} = 4 + 2x$$

$$2x + 4 = 2x + 4$$

$$\boxed{\text{INFINITELY MANY SOLNS}}$$

$$3(4x - 1) - 13x = -(x + 3)$$

$$12x - 3 - 13x = -x - 3$$

$$-x - 3 = -x - 3$$

$$\boxed{\text{INFINITELY MANY SOLNS}}$$

$$4(2x - 3) = 3(3x - 3) - 3$$

$$8x - 12 = 9x - 9 - 3$$

$$8x - 12 = 9x - 12$$

$$\boxed{1 \text{ solution} \rightarrow x = 0}$$

$$\frac{1}{4}(3 + 72x) = 6\left(3x + \frac{1}{4}\right)$$

$$\frac{3}{4} + 18x = 18x + \frac{6}{4}$$

$$\boxed{\text{NO SOLUTIONS!!}}$$