Study Guide: Equivalent Expressions

COMBINING LIKE TERMS

**Helpful Hints**

- As much as possible, change subtraction into addition by adding the inverse before combining like terms.
- Rearrange your terms to group terms that have the same variables closer to one another.
- When you combine like terms involving subtraction and parentheses, you need to multiply each of the terms inside the parentheses by $-1$.

*Example:*

$$4x + 3 - (3x + 12)$$

$$= 4x + 3 + (-1)(3x + 12)$$

- When you evaluate expressions by substituting numbers for variables, it will often be easier to combine like variable terms before substituting numbers.

*Example: When presented with the following expression:*

$$-4x - 3y - (-2x - 2y) + 5x - 11y$$

and are asked to evaluate when $x = 2$ and $y = -3$, you will want to simplify as variables before substituting.

- If working with fractions, it will be very helpful to convert all mixed numbers to improper fractions before doing any substantial work. For more info on how to convert from mixed numbers to improper fractions, see below.
- When combining like terms involving fractions, remember that all addition and subtraction needs to be done with fractions that have ***COMMON DENOMINATORS***. For more info on finding common denominators, see below.

**DISTRIBUTION**

This allows you to rewrite expressions that involve multiplication and addition or subtraction without changing the value of the expression.

*Example:*

$$4(3x + 2y + 6) = 12x + 8y + 24$$

We can also use array models to show distribution

*Example:*

<table>
<thead>
<tr>
<th>3x</th>
<th>2y</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>
**FACTORIZING**

Factoring is the opposite of distribution. You can create an equivalent with two or more terms that have a common factor.

*Example:* $4x + 10y + 14 = 2(2x + 5y + 7)$ We factor 2 out of each term and create an equivalent expression.

**Helpful Hints:**
- You can always use a factor tree to find the greatest common factor that all terms share. This intermediate step can be very helpful.
- If you are factoring where the term with the leading (first) variable has a negative coefficient, you need to factor out a negative number.

*Example:* $-6x + 9 = -3(2x + (-3)) = -3(2x - 3)$

**Array Models** can also show factoring:

```
? 4x 10y 14
```

**REVIEW OF FRACTION CONCEPTS**

**FIND A COMMON DENOMINATOR** When you are **ADDING AND SUBTRACTING**!

Example:

\[
\frac{1}{3} + \frac{5}{8}
\]

The smallest common denominator is 24. In order to add, we need both denominators to be 24. We can multiply by a fraction equal to 1:

\[
\left(\frac{1 \cdot 8}{3 \cdot 8}\right) + \left(\frac{5 \cdot 3}{8 \cdot 3}\right) = \frac{8}{24} + \frac{15}{24} = \frac{8 + 15}{24} = \frac{23}{24}
\]

**CONVERTING FROM A MIXED NUMBER TO AN IMPROPER FRACTION**
- This will make performing operations much easier for you.

\[
3 \frac{5}{8} = \frac{24}{8} + \frac{5}{8} = \frac{29}{8}
\]

(Because 3 is equal to $\frac{24}{8}$)
**STUDY GUIDE REVIEW ASSIGNMENT: DUE NOVEMBER 6!!**

Please take your time in looking over this review document. These problems are similar to the types of problems you will find on our test. I will answer questions based on this study guide on Wednesday November 5 in class. Please come prepared with questions. These questions are due to me before we take our exam and will count as a double homework assignment!

### Combine Like Terms. Evaluate the expression when \( x = -2 \) and \( y = 3 \).

\[
-3x + 4y - 12x - y
\]

\[
-3x + 4y + (-12x) + (-y)
\]

\[
(-3x + (-12x)) + (4y + (-y))
\]

\[
-15x + 3y
\]

(this is standard form)

\[
-15(-2) + 3(3)
\]

30 + 9

39

\[
-3x + (2y \cdot 5) - \left(\frac{1}{2}\right)x \cdot 4 + 7y
\]

**remember order of operations!!**

\[
-3x + 10y - 2x + 7y
\]

\[
-3x + 10y + (-2x) + 7y
\]

\[
(-3x + (-2x)) + 10y + 7y
\]

\[
-5x + 17y
\]

(this is standard form)

\[
-5(-2) + 17(3)
\]

10 + 51

61

### Combine Like Terms

\[
4r - 12s + 15t - (s + 4r - 15t)
\]

\[
4r + (-12s) + 15t + (-s) + (-4r) + 15t
\]

\[
(4r + (-4r)) + ((-12s) + (-s)) + (15t + 15t)
\]

\[
-13s + 30t
\]

\[
6m - 12n - 31p - (11p + 2n - 5m)
\]

\[
6m + (-12n) + (-31p) + (-11p) + (-2n) + 5m
\]

\[
(6m + 5m) + ((-12n) + (-2n)) + ((-31p) + (-11p))
\]

\[
11m + (-14n) + (-42p)
\]

\[
11m - 14n - 42p
\]
Emma wanted to help pay for her family’s gym membership so she could use it too. She paid $40 out of the $95 enrollment fee, and then she agreed to pay $\frac{1}{3}$ of the $87 monthly fee. Write an expression that represents how much money Emma’s family needs to pay towards their gym membership. Use $m$ for months.

\[
\left( m - \frac{1}{3}m \right) + (95 - 40)
\]

\[
\frac{2}{3}m + 55
\]

Create an equivalent expression by distributing.

\[
-7(2x + 7y - z)
\]

\[
-14x + (-49y) - (-7z)
\]

\[
-14x - 49y + 7z
\]

\[
\frac{1}{4} (8a - 12b - 28)
\]

\[
2a - 3b - 7
\]

<table>
<thead>
<tr>
<th>9g</th>
<th>5h</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>63g</td>
<td>35h</td>
</tr>
<tr>
<td>84</td>
<td>63g</td>
<td>35h</td>
</tr>
</tbody>
</table>

\[
-9(3x - 2y) - 3(6x - 4y)
\]

\[
-9(3x + (-2y)) - 3(6x + (-4y))
\]

\[
-27x + 18y + (-18x) + 12y
\]

\[
(-27x + (-18x)) + (18y + 12y)
\]

\[
-45x + 30y
\]
Create Equivalent Expressions by Factoring.

<table>
<thead>
<tr>
<th>$-18f - 9g + 12$</th>
<th>$32 - 24x - 16y - 8z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-3(6f + 3g - 4)$</td>
<td>$-24x - 16y - 8z + 32$</td>
</tr>
<tr>
<td></td>
<td>$-8(3x + 2y + z + 4)$</td>
</tr>
</tbody>
</table>

$-30d + 24f - 18d - 12f + 6d$

$-30d + 24f + (-18d) + (-12f) + 6d$

$(-30d + (-18d) + 6d) + (24f + (-12f))$

$-42d + 12f$

$-6(7d - 2f)$

<table>
<thead>
<tr>
<th>Factor the following expression inside the array model by filling in the question marks:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2v$</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>$3$</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

6 teachers each drank 3 cups of coffee and had 2 muffins each for breakfast on Monday. Write 2 equivalent expressions that show how much the teachers ate for breakfast. Use $c$ as a variable for coffee and $m$ for muffins. What do the different expressions represent?

Expression #1: $6(3c + 2m)$
This shows that each teacher had 3 cups of coffee and 2 muffins, and that there were 6 teachers.

Expression #2: $18c + 12m$
This shows that all together the teachers had 18 cups of coffee and a dozen muffins.

Mr. Mulkey went shopping for Halloween candy. He purchased the same amount of candy for each of his 3 classes. In all, he purchased 15 bags of Skittles and 9 bags of M&Ms. Write two equivalent expressions that show what Mr. Mulkey got for his students. Use $s$ for Skittles and $m$ for M&Ms. What do the different expressions represent?

Expression #1: $3(15s + 9m)$
This expression means that each class got 15 bags of skittles and 9 bags of M&Ms. Also, there were 3 classes total.

Expression #2: $45s + 27m$
This expression shows that all together, Mr. Mulkey bought 45 bags of skittles, and 27 bags of M&Ms. His students reported stomach aches within 15 minutes of class beginning.
Write an equivalent expression in standard form (simplify!) EXTRA CREDIT!!

\[
\frac{5}{6} (4x - 3) - \frac{3}{4} x - \frac{3(2x - 6)}{2} = \frac{20}{6} x - \frac{15}{6} - \frac{3}{4} x - \frac{6x - 18}{2}
\]
\[
20x + \left( -\frac{15}{6} \right) + \left( -\frac{3}{4} x \right) + (-3x) + 9
\]
\[
20x + \left( -\frac{3}{4} x \right) + (-3x) + \left( -\frac{15}{6} \right) + 9
\]
\[
\frac{40}{12} x + \left( -\frac{9}{12} x \right) + \left( -\frac{36}{12} x \right) + \left( -\frac{30}{12} \right) + \frac{108}{12}
\]
\[
-\frac{5}{12} x + \frac{78}{12}
\]
\[
-\frac{5}{12} x + \frac{13}{2}
\]

\[
\frac{(r - s)}{3} - \frac{(3s - 2r)}{5} + \frac{11}{10} r
\]
\[
\left( \frac{10}{10} \right) \frac{(r - s)}{3} - \left( \frac{6}{6} \right) \frac{(3s - 2r)}{5} + \left( \frac{3}{3} \right) \frac{11}{10} r
\]
\[
\frac{10(r - s)}{30} - \frac{6(3s - 2r)}{30} + \frac{33r}{30}
\]
\[
\frac{10r - 10s - 18s + 12r + 33r}{30}
\]
\[
\frac{10r + (-10s) + (-18s) + 12r + 33r}{30}
\]
\[
\frac{(10r + 12r + 33r) + ((-10s) + (-18s))}{30}
\]
\[
\frac{55r + (-28s)}{30}
\]
\[
\frac{55r - 28s}{30}
\]
\[
\begin{align*}
6 \frac{1}{3} x - \left( 3 \frac{2}{3} \right) \left( \frac{1}{2} - \frac{3}{4} x \right) &= \frac{19}{3} x - \left( \frac{11}{3} \right) \left( \frac{1}{2} - \frac{3}{4} x \right) \\
&= \frac{19}{3} x - \left( \frac{11}{3} \right) \left( \frac{1}{2} + \left( -\frac{3}{4} x \right) \right) \\
&= \frac{19}{3} x + \left( -\frac{11}{6} \right) + \frac{33}{12} x \\
&= \frac{76}{12} x + \left( -\frac{11}{6} \right) + \frac{33}{12} x \\
&= \left( \frac{76}{12} x + \frac{33}{12} x \right) + \left( -\frac{11}{6} \right) \\
&= \frac{109}{12} x - \frac{11}{6}
\end{align*}
\]

\[
\begin{align*}
\frac{3}{4} x - \frac{7}{10} x + \frac{3}{5} x - 2x &= \frac{3}{4} x + \left( -\frac{7}{10} x \right) + \frac{28}{5} x + ( -2x ) \\
&= \left( \frac{5}{4} x + \frac{2}{2} \left( -\frac{7}{10} x \right) + \frac{4}{4} \frac{28}{5} x + \frac{20}{20} (-2x) \right) \\
&= \frac{15}{20} x + \left( -\frac{14x}{20} \right) + \frac{112}{20} x + \left( -\frac{40}{20} \right) x \\
&= \frac{15x + (-14x) + 112x + (-40x)}{20} \\
&= \frac{73}{20} x
\end{align*}
\]