

PROPORTIONAL RELATIONSHIPS, RATIOS AND UNIT RATES STUDY GUIDE

UNIT RATES

A **ratio** is an ordered pair of non-negative numbers, denoted as A:B. A is the first number and B is the second number.

Example: There are 22 girls and 11 boys in this class. The ratio is written as 22:11.

A **rate** is a ratio of two quantities. This is written as a fraction.

Example: The rate of girls per boys in our class is $\frac{\text{girls}}{\text{boys}} = \frac{22}{11}$.

A **unit rate** is the value of the rate—that is the value of the fraction above where the denominator is 1.

Example: The unit rate of girls to boys is 2. This means there are 2 girls for every 1 boy. Boys are in the denominator.

TIP: A unit rate will usually include the words “per” or “for each” or “for every.” This is how you know you’re being asked to find a unit rate. Also, the quantity that comes after this is usually in the denominator.

Example: 2 pounds of rice cost \$1.50. How much is rice per pound? The rate is measured as $\frac{\text{Cost (in \$)}}{\text{Rice (in lbs)}} = \frac{\$1.50}{2}$. The unit rate is \$0.75. This means that rice cost \$0.75 per pound.

PROPORTIONAL RELATIONSHIPS

Two quantities are proportional to each other if there is one constant number (called the **constant of proportionality**, denoted by k) that is multiplied by each measure in the first quantity to give the corresponding measure in the second quantity. Proportional Relationships can be viewed as tables, graphs, and equations.

Example for Page 2 and 3:

Self Serve Yogurt cost \$0.40 per ounce

PROPORTIONAL RELATIONSHIPS AND TABLES

Yogurt (in ounces)	Price (in dollars)
1	0.40
6	2.40
7.5	3.00
	6.40
21	

For every ounce of yogurt, you will pay \$0.40. In this case, the constant of proportionality is \$0.40. If you are given the amount of yogurt, you need to multiply by \$0.40 to find out the total price. If you are given the total price, you need to divide by \$0.40 to find how many ounces of yogurt you could get for that price.

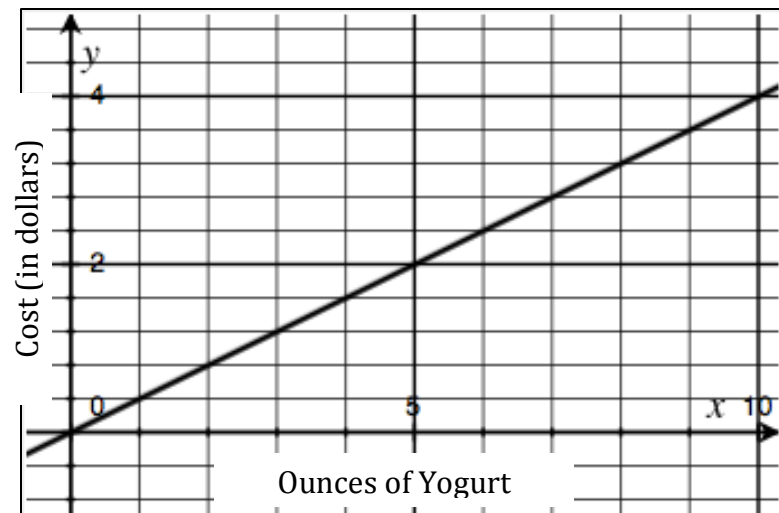
PROPORTIONAL RELATIONSHIPS AND GRAPHS

Graphs of proportional relationships are always straight lines and they always pass through the origin (0,0).

The **x-axis** is the horizontal axis. It goes side to side.

The **y-axis** is the vertical axis. It goes up and down.

Coordinate points are always read as (x, y) , the horizontal measurements are read first, then the vertical measurements.



The graph above shows the proportional relationship of ounces of yogurt (on the x-axis) and the cost (on the y-axis).

PROPORTIONAL RELATIONSHIPS AND EQUATIONS

We use the following equation to represent proportional relationships:

$$y = kx$$

***k* is the constant of proportionality. It is also the SAME THING as the unit rate.**

Example: The equation to represent the cost of yogurt per ounce is $y = 0.4x$

In order to find the constant of proportionality if you are provided with a rate, you can use the following equation:

$$k = \frac{y}{x}$$

TIP: A closer look at independent and dependent variables. Proportional relationships contain independent and dependent variables. The amount of the dependent variable literally relies on the amount of the independent variable.

Example: In our yogurt example, this means the price you pay for yogurt depends on how much you get. Therefore the price is dependent and the amount is independent.

Independent Variable	Dependent Variable
This is the x-value on a graph.	This is the y-value on a graph.
This is the first column in a table	This is the second column in a table.
When you are calculating a rate, the independent variable is the denominator	When you are calculating a rate, the dependent variable is the numerator
If time is one of the quantities, it is often the independent variable.	If money is one of the quantities, it is often the dependent variable.

EXERCISES

Please complete the following exercises and submit this at the same time you take your assessment. This is due on September 15, 2016

Calculate the unit rate in the following scenarios. Write an equation in $y = kx$ form to represent the proportion.

<p>A recipe that would make 18 cupcakes calls for 4 teaspoons of vanilla. How many cupcakes is that per teaspoon?</p> <p>To solve this problem, we set up a proportion as follows:</p> $\frac{\text{cupcakes}}{\text{vanilla}} = \frac{18}{4} = 4.5$ <p>That means for every 4.5 cupcakes, we need one teaspoon of vanilla.</p>	<p>Refer to the question to the left. How would you figure out how many teaspoons of vanilla you'd need for to make one cupcake?</p> <p>This is the inverse of the question on the left, and we set up the following proportion:</p> $\frac{\text{vanilla}}{\text{cupcakes}} = \frac{4}{18} = \frac{2}{9}$ <p>This means you can make one cupcake using $\frac{2}{9}$ of a teaspoon of vanilla.</p>
<p>You send 1600 texts each month and pay \$40 for this service. How much money is that per text?</p> <p>To solve this problem you figure out how much one text cost, this is</p> $\frac{\text{cost}}{\text{texts}} = \frac{40}{1600} = 0.025$ <p>This is 2.5 cents per text.</p>	<p>Refer to the question to the left. How many texts can you send for \$1?</p> <p>I can send 40 texts because each is 2.5 cents...and 2.5 cents times 40 is a dollar</p>
<p>Ms. Galassi can run 8.5 miles in 2 hours. What is her pace in miles per hour?</p> <p>4.25 miles an hour</p> <p>How long will it take her to run 14 miles?</p> <p>This is a difficult problem. You can build a table of values or divide $14 \div 4.25$ to find out that Ms. Galassi will need about 3 hours and 18 minutes to run 14 miles.</p>	<p>Mr. Rogove runs 12 miles in 2 hours and 30 minutes. Is he faster or slower than Ms. Galassi? What is his pace in miles per hour?</p> <p>Of course, Mr. Rogove is faster...he's going at 4.8 miles an hour. And this is after two knee surgeries. To find out his rate, we need to divide $12 \div 2.5$ to get our answer of 4.8 MPH.</p> <p>Honestly, Mr. Rogove can probably go about 7.5MPH.</p>

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Calculate the unit rate by examining the following tables. Fill in the missing data and write an equation in $y = kx$ form to represent the proportion.

<table border="1" style="width: 100%; border-collapse: collapse; margin-bottom: 10px;"> <thead> <tr> <th style="padding: 5px;">Amount of Mushroom (lbs)</th> <th style="padding: 5px;">Price paid (in dollars)</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">3</td> <td style="text-align: center;">5.25</td> </tr> <tr> <td style="text-align: center;">5</td> <td style="text-align: center; color: red;">8.75</td> </tr> <tr> <td style="text-align: center;">6</td> <td style="text-align: center; color: red;">10.50</td> </tr> <tr> <td style="text-align: center; color: red;">12</td> <td style="text-align: center;">21.00</td> </tr> <tr> <td style="text-align: center; color: red;">19</td> <td style="text-align: center;">33.25</td> </tr> </tbody> </table> <p style="color: red;">$y = 1.75x$ This means that mushrooms cost \$1.75 per pound.</p>	Amount of Mushroom (lbs)	Price paid (in dollars)	3	5.25	5	8.75	6	10.50	12	21.00	19	33.25	<table border="1" style="width: 100%; border-collapse: collapse; margin-bottom: 10px;"> <thead> <tr> <th style="padding: 5px;">Amount of sugar (cups)</th> <th style="padding: 5px;">Number of brownies made</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">3</td> <td style="text-align: center;">10</td> </tr> <tr> <td style="text-align: center;">6</td> <td style="text-align: center; color: red;">20</td> </tr> <tr> <td style="text-align: center;">10</td> <td style="text-align: center; color: red;">$33.\bar{3}$</td> </tr> <tr> <td style="text-align: center; color: red;">7.5</td> <td style="text-align: center;">25</td> </tr> <tr> <td style="text-align: center;">15</td> <td style="text-align: center; color: red;">50</td> </tr> </tbody> </table> <p style="color: red;">$y = \frac{10}{3}x$. This means that for every $\frac{10}{3}$ brownies you make, you'll need 1 cup of sugar.</p>	Amount of sugar (cups)	Number of brownies made	3	10	6	20	10	$33.\bar{3}$	7.5	25	15	50
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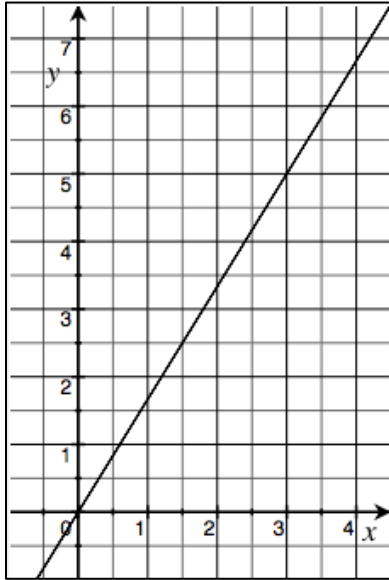
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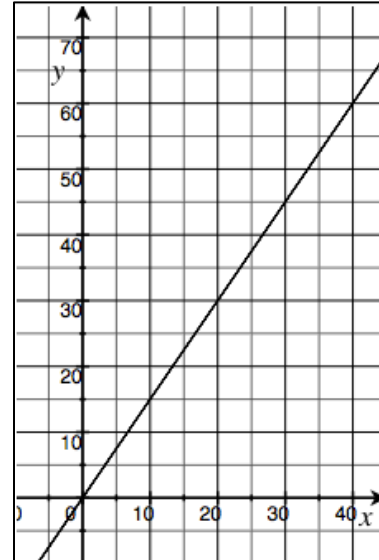
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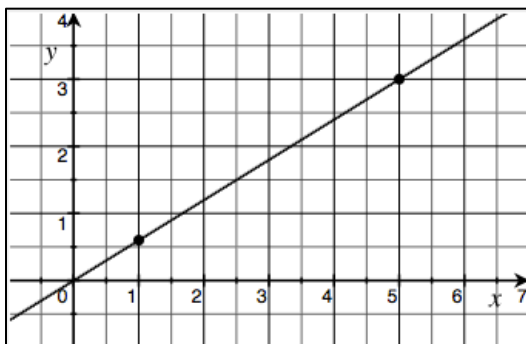
Calculate the unit rate by examining the following graphs. Write an equation in $y = kx$ form to represent the proportion.



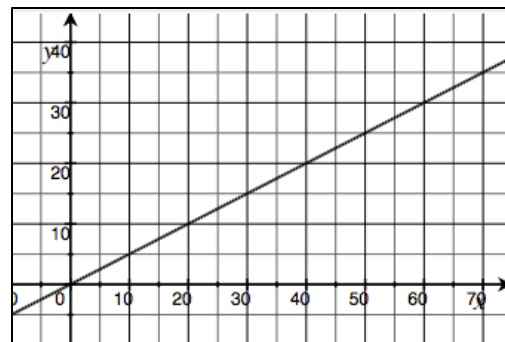
Since this passes through the origin and the point (3, 5), the equation that would represent this proportion is $y = \frac{5}{3}x$.



Since this passes through the origin and the point (10,15), the equation that would represent this proportion is $y = \frac{3}{2}x$.



Since this passes through the origin and the point (5,3), the equation that would represent this proportion is $y = \frac{3}{5}x$.



Since this passes through the origin and the point (10,5), the equation that would represent this proportion is $y = \frac{1}{2}x$.

I am wanting to make a purple blend of paint for Molly's bedroom. I realize that the blend I want requires 3 quarts of red paint and 4 quarts of blue paint. If I had $10\frac{1}{2}$ quarts of red paint, how many quarts of blue paint would I need? If I had 6 quarts of blue paint, how many quarts of red paint would I need?

I think it will help to make a table of values...

Red Paint	Blue Paint
3	4
6	8
9	12
12	16

For the red paint, since $10\frac{1}{2}$ is halfway between 9 and 12, the amount of blue paint is halfway in between 12 and 16...in other words, I'd need 14 quarts of blue paint if I had $10\frac{1}{2}$ quarts of red paint.

For the blue paint, since 6 is halfway between 4 and 8, the amount of red paint is half way between 3 and 6...in other words, I'd need $4\frac{1}{2}$ quarts of red paint if I had 6 quarts of blue paint.

Additionally, since this is a proportional relationship, I could also use an equation with the constant of proportionality being $\frac{4}{3}$, and the equation being $y = \frac{4}{3}x$.

I recently purchased a long piece of wood. The piece of wood was 16 feet long. I needed to cut as many smaller pieces that are $1\frac{1}{3}$ feet long as possible. How many smaller pieces can I get from that long board?

I want to cut the smaller pieces that are $1\frac{1}{3}$ feet long and I have 16 feet all together. There are a few things I can do...I can make a table again...

Small pieces of wood	Total length (in feet)
1	$1\frac{1}{3}$
2	$2\frac{2}{3}$
5	$6\frac{2}{3}$
10	$13\frac{1}{3}$
12	16

Since 10 boards are $13\frac{1}{3}$ feet and 2 boards are $2\frac{2}{3}$ feet, this equals 16 feet, which means I can cut 12 smaller boards from the 16 foot piece.

I also could have divided—this probably would be simpler:

$$16 \div \frac{4}{3}$$

$$= 16 \times \frac{3}{4} = 12$$

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