Proportional Relationships, Ratios and Unit Rates Study Guide

Proportional Relationships

Two quantities are proportional to each other if there is one constant number (called the **constant of proportionality**, denoted by **k**) that is multiplied by each measure in the first quantity to give the corresponding measure in the second quantity. Proportional Relationships can be viewed as tables, graphs, and equations.

Proportional Relationships and Tables

For every ounce of yogurt, you will pay $0.40. In this case, the constant of proportionality is $0.40. If you are given the amount of yogurt, you need to multiply by $0.40 to find out the total price. If you are given the total price, you need to divide by $0.40 to find how many ounces of yogurt you could get for that price.

| Yogurt (in ounces)	1	2	6	7.5	12		21
Price (in dollars)	0.40	0.80	2.40	3.00	4.80	6.40	

Proportional Relationships and Graphs

Graphs of proportional relationships are always straight lines and they always pass through the origin (0,0).

The **x-axis** is the horizontal axis. It goes side to side.

The **y-axis** is the vertical axis. It goes up and down.

Coordinate points are always read as (x, y), the horizontal measurements are read first, then the vertical measurements.
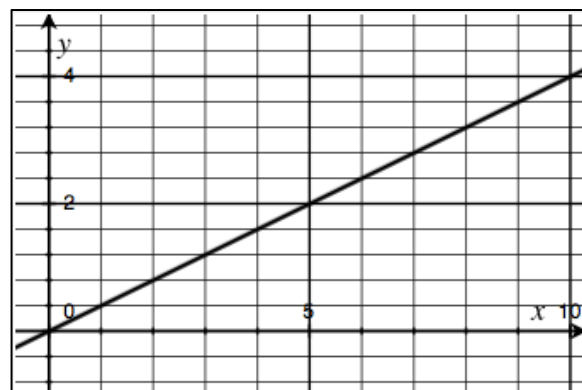
The graph above shows the proportional relationship of ounces of yogurt (on the *x*-axis) and the cost (on the *y*-axis).

PROPORTIONAL RELATIONSHIPS AND EQUATIONS

We use the following equation to represent proportional relationships:

$$y = kx$$

k is the constant of proportionality. It is also the SAME THING as the unit rate.

Example: The equation to represent the cost of yogurt per ounce is $y = 0.4x$

In order to find the constant of proportionality if you are provided with a rate, you can use the following equation:

$$k = \frac{y}{x}$$

TIP: A closer look at independent and dependent variables. Proportional relationships contain independent and dependent variables. The amount of the dependent variable literally relies on the amount of the independent variable.

Example: In our yogurt example, this means the price you pay for yogurt depends on how much you get. Therefore the price is dependent and the amount is independent.

Independent Variable	Dependent Variable
This is the x-value on a graph.	This is the y-value on a graph.
This is the first column in a table	This is the second column in a table.
When you are calculating a rate, the independent variable is the denominator	When you are calculating a rate, the dependent variable is the numerator
If time is one of the quantities, it is often the independent variable.	If money is one of the quantities, it is often the dependent variable.

COMPLEX FRACTIONS

A **complex fraction** is a fraction that has a fraction in the numerator and/or denominator.

Examples:

$\dfrac{3\frac{2}{3}}{1\frac{4}{5}}$	$\dfrac{\frac{7}{2}}{\frac{14}{3}}$	$\dfrac{7}{\frac{4}{5}}$
Numerator and denominator can be written as mixed numbers	Numerator and denominator can be written as improper fractions	It's possible the numerator or denominator might NOT be a fraction.

To solve complex fractions, treat the fraction as a division problem. Remember that when you divide fractions you are really multiplying by the reciprocal.

Example:

$$\frac{\frac{7}{2}}{\frac{14}{3}} \;=\; \frac{7}{2} \div \frac{14}{3} \;=\; \mathbf{\frac{7}{2}} \times \mathbf{\frac{3}{14}} \;=\; \frac{21}{28} \;=\; \frac{3}{4}$$

TIP: Remember that you might need to convert units before doing some calculations.

Examples:
- There are 16 ounces in a pound. 21 ounces is NOT 2.1 pounds. It is 1 pound and 5 ounces, or $1\frac{5}{16}$ pounds.
- There are 60 minutes in an hour. 2 hours and 25 minutes is NOT $2\frac{1}{4}$ hours. It is $2\frac{25}{60}$ hours or $2\frac{5}{12}$ hours
- There are 12 inches in a foot. 6 feet and 6 inches is NOT 6.6 feet. It is $6\frac{6}{12}$ feet or $6\frac{1}{2}$ feet.

EXERCISES

Please complete the following exercises and submit this at the same time you take your assessment. This is due on September 29, 2016.

Calculate the unit rate in the following scenarios. Write an equation in $y = kx$ form to represent the proportion.

Ms. Tran bought 2.5 pounds of stew meat for her famous beef stew. She paid $14.95 for the meat? What is the unit rate price per pound? Write an equation to represent this. $$\frac{price}{pounds} = \frac{14.95}{2.5} = 5.98 \text{ per pound}$$ Since the unit rate is 5.98, the equation is $y = 5.98x$	Ms. Tran's recipe says that it feeds 5 people. If Ms. Tran expects to have 12 people over to dinner, how much stew meat does she need to buy? How much will this cost her? If Ms. Tran spends $14.95 to feed 5 people, how much does it cost to feed one person?? $\frac{14.95}{5} = 2.99$. Since it cost $2.99 per person, and she needs to feed 12 people, multiply: $12 \times 2.99 = \$35.88$ It will cost $35.88 to feed 12 people.
If 5/6 lb. of candy costs $15.50. How much would 1 lb. of the same candy cost? You can use a tape diagram to solve this, or math…if 5/6 of a pound cost $15.50, simply divide by 5 to figure out how much 1/6 of a pound cost, and then multiply by 6 to figure out how much one entire pound costs. $$\frac{15.50}{5} = 3.10$$ $$6 \times 3.10 = 18.60$$ Each pound of candy costs $18.60	If Ms. Galassi only had $4.65, how much candy would she able to buy? Ms. Galassi would be able to buy only ¼ of a pound of candy. $$\frac{4.65}{18.60} = \frac{1}{4}$$

Ms. Galassi can run 8.75 miles in 1 hour and 45 minutes. What is her pace in miles per hour?

To solve this, find the unit rate:
$$\frac{8\frac{3}{4}}{1\frac{3}{4}} = \frac{\frac{35}{4}}{\frac{7}{4}} = 5$$ Ms. Galassi is running at 5 MPH.

How long will it take her to run 14 miles?

She will run 14 miles in more than 2 hours, but not quite 3 hours. To solve this, divide. $\frac{14}{5} = 2.8$ It will take Ms. Galassi 2 hours and 48 minutes (48 minutes is 8/10 of an hour).

Mr. Rogove runs 12 miles in 2 hours and 30 minutes. Is he faster or slower than Ms. Galassi? What is his pace in miles per hour?

Mr. Rogove's rate can be calculated as follows: $\frac{12}{2.5}$, which is 4.8 miles an hour. He is slower than Ms. Galassi.

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Tran/Rogove   Date:_____

Calculate the unit rate by examining the following tables. Fill in the missing data and write an equation in $y = kx$ form to represent the proportion.

Amount of Mushroom (lbs)	Price paid (in dollars)
0.75	5.25
1.25	8.75
3	21
5.25	36.75
6	42.00

$\frac{5.25}{0.75} = 7$ The unit rate is 7, and the equation is $y = 7x$. To find unknown prices, multiply mushroom pounds by 7. To find unknown mushroom amounts, divide the price by 7.

Amount of sugar (cups)	Number of brownies made
$\frac{10}{9}$	10
2	18
$2\frac{7}{9}$	25
$4\frac{1}{3}$	39
6	54

$\frac{25}{2\frac{7}{9}} = 9$ the unit rate is 9, and the equation is $y = 9x$. To find unknown brownies, multiply cups of sugar by 9. To find unknown cups of sugar, divide brownies by 9.

# of songs on the radio	Time spent listening
4	14 minutes, 24 seconds
7	25 min 12 sec
10	36 minutes
15	54 minutes
31	1 hour, 51 minutes, 36 seconds

$\frac{14\frac{2}{5}}{4} = 3.6$ This means each song is 3.6 minutes long. .6 of a minute 36 seconds....so the equation is $y = 3.6x$.

**assume all songs are the same length!
**Time is NOT independent this time. It's dependent on the number of songs you're listening to.

Time	Distance walked
50 minutes	$\frac{9}{8}$ miles
60 minutes	$\frac{27}{20}$ miles (1.35)
90 minutes	$\frac{81}{40}$ miles (2.025)
150 minutes	$3\frac{3}{8}$ miles
240 minutes (4 hrs)	$5\frac{2}{5}$ miles

50 minutes is $\frac{5}{6}$ of an hour. To find miles per hour, set up the following ratio:

$\frac{\frac{9}{8}}{\frac{5}{6}}$ and you find that you can walk

$\frac{27}{20}$ or 1.35 miles per hour. Either multiply or divide by $\frac{27}{20}$ depending on your unknown.

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Simplify the following expressions.

$$\dfrac{20\frac{1}{2}}{4\frac{1}{6}}$$	$$\dfrac{\frac{4}{5}}{6\frac{2}{3}}$$
$$\dfrac{\frac{41}{2}}{\frac{25}{6}} = \frac{41}{2} \div \frac{25}{6}$$	$$\dfrac{\frac{4}{5}}{\frac{20}{3}} = \frac{4}{5} \div \frac{20}{3}$$
$$= \frac{41}{2} \times \frac{6}{25}$$	$$= \frac{4}{5} \times \frac{3}{20}$$
$$= \frac{246}{50} = \frac{123}{25}$$	$$= \frac{12}{100} = \frac{3}{25}$$
I am wanting to make a purple blend of paint for Molly's bedroom. I realize that the blend I want requires $3\frac{1}{4}$ quarts of red paint and $2\frac{1}{2}$ quarts of blue paint. If I had 1 gallon of red paint, how many gallons of blue paint would I need? If I had one gallon of blue paint, how many gallons of red paint would I need? Tough question…First, let's find out how many quarts of blue I'd need for 1 quart of red…and then multiply that result by 4 (because there are 4 quarts in a gallon) $$\frac{2\frac{1}{2}}{3\frac{1}{4}} = \frac{\frac{5}{2}}{\frac{13}{4}}$$ $$= \frac{5}{2} \div \frac{13}{4} = \frac{5}{2} \times \frac{4}{13} = \frac{20}{26}$$ $$= \frac{10}{13} \text{ so, multiply } \frac{10}{13} \times 4$$ $$= \frac{40}{13} \text{ or } 3\frac{1}{3} \text{ gallons}$$	I recently purchased a long piece of wood (for my table saw!). The piece of wood was 12 feet 6 inches long. I needed to cut as many smaller pieces that are $1\frac{9}{16}$ feet long as possible. How many smaller pieces can I get from that long board? 12 feet 6 inches is $12\frac{1}{2}$ feet….we need to divide… $$\dfrac{12\frac{1}{2}}{1\frac{9}{16}} = \dfrac{\frac{25}{2}}{\frac{25}{16}} = \frac{25}{2} \div \frac{25}{16}$$ $$= \frac{25}{2} \times \frac{16}{25}$$ $$= \frac{400}{50} = 8$$

Study Guide: Proportional Relationships, Ratios, and Unit Rates

Maia and Kaishu are working on the following problem involving proportions:

Erubiel went on a long bike ride. He rode for 2 hours and 15 minutes, and traveled 43 miles. What is his rate?

Maia said the answer was $19\frac{1}{9}$ miles per hour. Kaishu said the answer 20 miles per hour.

a. What work might Maia have done to get her an answer of $19\frac{1}{9}$? Is this correct? Explain your answer.

2 hours and 15 minutes is 2.25 hours…$2\frac{1}{4}$ hours…so let's divide.

$$\frac{43}{2\frac{1}{4}} = \frac{43}{1} \div \frac{9}{4}$$

$$= \frac{43}{1} \times \frac{4}{9}$$

$$= \frac{172}{9} = 19\frac{1}{9}$$

Maia is correct. She figured out that 2 hr 15 min are 2.25 hours and divided.

b. What work might Kaishu have done to get his answer of 20? Is this correct? Explain your answer.

Kaishu was incorrect…he likely thought that 2 hours and 15 minutes was 2.15 hours, and divided accordingly.

$$\frac{43}{2.15} = 20$$

Silly Kaishu. Be like Maia.