

LEARNING OBJECTIVE: We will understand the concept of exponential decay and solve problems involving exponential decay. (Alg1M3L6)

ACTIVATING PRIOR KNOWLEDGE

We can identify problems involving exponential growth.

Ryan deposits \$200 in his bank account. Due to a crazy bank error, each day his deposit grows. After 1 week, he has \$300 in his account, After 2 weeks, he has \$450. After three weeks, he checks his account and he has \$675.

What is the formula that models this problem?

$$R(t) = 200(1.5)^t$$

How do you know this involves exponential growth?

$$b > 1$$

How much will he have after four weeks?

$$R(4) = 200(1.5)^4 = \$1012.50$$

Malia is at the nursery picking up seedlings for summer. She buys a magic bean seed when it's 1 foot tall. After 1 month, she measures it and it's 2 feet 6 inches. After 2 months, it's 6 feet 3 inches, and after 3 months, it's 15 feet 7.5 inches.

What is the formula that models this problem?

$$f(x) = 12(2.5)^x$$

How do you know this involves exponential growth?

$$2.5 > 1$$

$$200 \times _ = 300$$

$$300 \times _ = 450$$

$$450 \times _ = 675$$

$$12 \text{ ft} \\ 30 \\ 75 \\ 187.5$$

CONCEPT DEVELOPMENT

When you buy a new car, the value drops considerably when the driver takes the car off the lot. Some cars will lose 15% of their value when you drive off the lot, and continue to lose 15% of their value each year. Below, we will complete a chart modeling this situation for a new car costing \$15,000.

Number of years (t) passed since driving car off lot	Car value after t years	15% depreciation of current car value	Car value minus 15% depreciation
0	\$12,750.00	\$1,912.50	\$10,837.50
1	10,837.50	1,625.63	\$9,211.87
2			
3			
4			\$5,657.24

$$V(t) = 15000(.85)^{t+1}$$

$$(1 - .15) = .85$$

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Exponential Decay is similar to exponential growth except the growth factor is between 0 and 1. Over time, the output of an exponential decay function will diminish.

Exponential decay functions will be in the form: $f(x) = ab^x$ where $0 < b < 1$.
Exponential growth functions will ALSO be in the form: $f(x) = ab^x$ but $b > 1$

Practice Identify the initial value and state whether the formula is modeling exponential growth or decay, and justify your answer.

$f(t) = \frac{1}{2} \left(\frac{2}{5} \right)^t$ $a = 2 \quad \left. \begin{array}{l} \text{Decay} \\ b = \frac{2}{5}, \frac{2}{5} < 1 \end{array} \right\}$	$f(t) = 2 \left(\frac{5}{3} \right)^t$ $a = 2 \quad \left. \begin{array}{l} \text{growth} \\ b = \frac{5}{3}, \frac{5}{3} > 1 \end{array} \right\}$
$f(t) = \frac{2}{3} (3)^t$ $a = \frac{2}{3} \quad \left. \begin{array}{l} \text{Growth} \\ b = 3, 3 > 1 \end{array} \right\}$	$f(t) = \frac{2}{3} \left(\frac{1}{3} \right)^t$ $a = \frac{2}{3} \quad \left. \begin{array}{l} \text{Decay} \\ 0 < \frac{1}{3} < 1 \end{array} \right\}$
$f(t) = \frac{3}{2} \left(\frac{2}{3} \right)^t$ $a = \frac{3}{2} \quad \left. \begin{array}{l} \text{Decay} \\ b = \frac{2}{3}, \frac{2}{3} < 1 \end{array} \right\}$	$f(t) = .95(1.01)^t$ $a = .95 \quad \left. \begin{array}{l} \text{growth} \\ b = 1.01 \\ 1.01 > 1.00 \end{array} \right\}$

GUIDED PRACTICE**Steps for Evaluating Exponential Decay Functions**

1. Identify the growth factor and the initial value of the function.
2. Answer the questions posed.

If a person takes a dosage (d) of a particular medication then the formula $f(t) = d(0.8)^t$ represents the concentration of medication in the bloodstream t hours later. If Anna takes 200mg of the medication at 6:00AM, how much remains in her bloodstream at 10:00AM. $t = \text{time}$ $d = \text{dosage}$

$$f(4) = 200(0.8)^4$$

$$200(.4096) \quad 81.92 \text{ mg.}$$

$f(t) = \text{amt of medication in blood}$
How long does it take for the concentration to drop below 1mg?

$$f(t) < 1 \quad 200 \cdot (0.8)^t < 1$$

24 hours

As people approach retirement, they begin to spend their life savings (s). One model for a particular retiree is $f(t) = s(0.95)^t$ represents the money a person has after t years of retirement. If Cindy has \$6,000,000 saved up when she retires, how much money will remain after she's been retired for 7 years?

$S = \text{life savings at retirement.}$
 $t = \text{years of retirement}$
 $f(t) = \text{amount of \$ you have after } t \text{ years of retirement.}$

$$f(7) = 6,000,000(0.95)^7 = \$4,190,023.78$$

How many years will pass before she is no longer a millionaire?

$$35 \text{ years.} \quad \$996,500.30$$

Ryan bought a new computer for \$2,100. The value decreases by 50% each year. Write the function that models this situation.

$$f(x) = 2100(.5)^x$$

When will the value drop below \$300?

$$f(x) < 300 \quad \underline{3 \text{ years}}$$

$$\frac{2100(.5)^x < 300}{2100 \quad 2100}$$

$x = 3$

Akshat's dad takes 400mg of ibuprofen. Each hour, the amount of ibuprofen in a person's system decreases by 29%. Write the function that models this situation.

~~$$f(t) = d(.29)^t \quad t \text{ (KAVI)}$$~~

$$f(t) = 400(.71)^t$$

How much ibuprofen is left in his system after 3 hours?

$$f(3) = 400(.71)^3$$

$$143.16 \text{ mg.}$$

A rubber ball is dropped from a height of 30 feet. On the first bounce, it bounces up to 22.5 feet. Each successive bounce goes 75% as high as the bounce before it. Write a function that models this situation.

After how many bounces will the ball bounce up under 1 foot?

An object attached to a bungee cord is released from a height of 60 feet. After the first bounce, the object goes back up to a height of 54 feet. After the second bounce, it reaches a height of 48.6 feet. After the third bounce, it reaches a height of 43.74 feet.

What is the growth factor of the function?

Write the function:

How high will the object go back up after 6 bounces?



CLOSURE

A huge ping pong tournament is held in Beijing with 65,536 participants at the start of the tournament. Each round of the tournament eliminates half of the participants.

a. If $p(r)$ represents the number of participants after r rounds of play, write a formula to model number of participants remaining.

$$\rightarrow p(r) = 65536 \left(\frac{1}{2}\right)^r \leftarrow$$

b. Use the model to determine how many participants remain after 10 rounds of play.

$$\begin{aligned} p(10) &= 65536 \left(\frac{1}{2}\right)^{10} \\ &= 65536 \left(\frac{1}{1024}\right) = 64 \end{aligned}$$

c. How many rounds will it take to crown a champion?

$$p(r) = 1 \qquad 65536 \left(\frac{1}{2}\right)^r = 1$$

16 rounds

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INDEPENDENT PRACTICE

Complete problem set from Lesson 7.

NOTES

MAPS TO LESSON 7, ALG 1 MOD 3