LEARNING OBJECTIVE: We will compare growth rates of linear and exponential models over intervals of equal length. (Alg1M3L7)

CONCEPT DEVELOPMENT:

Exponential Functions: Functions that are in the form $f(x) = ab^x$ $b^0 = 1$
For any exponential function in the form above, $f(0) = a$
The function grows multiplicatively by b over the interval of length 1.   *a is the starting amount*

Example: Joe has 4 Instagram followers to begin with, and each day the number grows as indicated on the table below.

Day x	0	1	2	3	4	5	6
Number of followers $f(x)$	4	8	16	32	64	128	256

Growth observed: FOLLOWERS DOUBLING. MULTIPLYING BY 2.

Formula for above: $f(0) = a = 4$ $b = 2$ $f(x) = 4 \cdot 2^x$
$f(x) = ab^x$

MAIN TAKEAWAY: Exponential Functions grow MULTIPLICATIVELY!

Linear Functions: Functions that are in the form $f(x) = mx + b$
For any linear function, $f(0) = b$
The functions grows additively by m over the interval length of 1.
Example: Molly begins with $150 in her bank account, and each week her account grows as indicated by the table below.

Week x	0	1	2	3	4	5
Bank Account Balance $f(x)$	150	155	160	165	170	175

Growth Observed: ADDING $5 EACH WEEK

Formula for above: $f(x) = mx + b$ $f(x) = 5x + 150$ $b = 150$ $m = 5$

MAIN TAKEAWAY: Linear Functions grow ADDITIVELY!

Practice:

A lab researcher records the growth of the population of a yeast colony and finds that the population doubles every hour.

a. Complete the table below

Hours Into Study	0	1	2	3	4
Yeast Colony Population (thousands)	5	10	20	40	80

b. What is the exponential function that models the growth of the colony's population?

$$f(x) = ab^x \qquad a = 5 \qquad f(x) = 5 \cdot 2^x$$
$$b = 2$$

c. Several hours into the study, the researcher looks at the data and wishes that there were more frequent measurements. If he knows the colony doubles every hour, how can he determine the population in half hour increments?

Let x = the factor by which the population grows every $\frac{1}{2}$ hr.

It will grow by another "x" in the second half hour.

In 1 hr, it grows by x^2 $x^2 = 2$ $x = \sqrt{2}$

d. Complete the new table that includes half hour increments.

Hours into Study	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3
Yeast Colony Population (thousands)	5	7.071	10	14.142	20	28.284	40

e. A research assistant studying the data thinks that since the population doubles every hour, that it will grow by half that in 30 minutes…and claims that $t\left(\frac{1}{2}\right) = 7.5$. Is the research assistant correct? Why/why not?

No! The function is exponential not linear

The rate of change is not the same

(handwritten, left margin) "0" year is 1900.

2. A California Population Projection Engineer in 1920 was tasked with finding a model that predicts the state's growth. He modeled the population growth as a function of time, t years, since 1900. Census data showed that the population in 1900, in thousands, was 1,490. In 1920, the population of California was 3,554 thousand. He decided to explore both a linear and exponential model.

a. Use the data provided to determine the equation of the linear function that models the population growth from 1900-1920. Label this function $f(t)$.

(handwritten) $f(t) = mt + b$ $f(t) = 103.2t + 1490$

(handwritten, right) $b = 1490$ $m = \dfrac{3554 - 1490}{20} = \boxed{103.2}$

b. Use the data provided and a calculator to determine the equation of the exponential function that models the population growth. Label this one $g(t)$.

(handwritten) $g(t) = ab^t$ $g(t) = 1490 \cdot b^t$ $g(0) = a = 1490$

$b^{20} = 2.385$
$3554 = 1490 \cdot b^{20}$ $b = 1.044$

(handwritten, right) $g(t) = 1490 \cdot 1.044^t$

c. Use the two functions to predict the population in the following years:
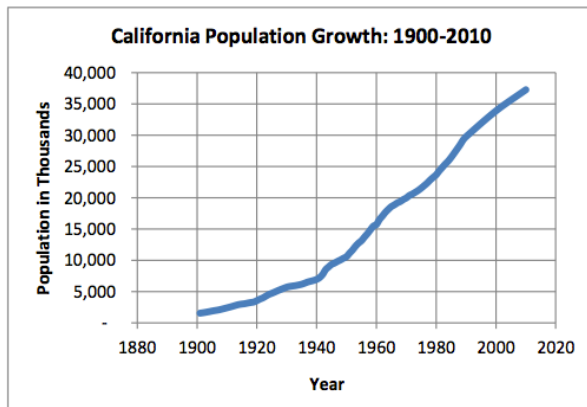
	Projected Population Based on Linear Function, $f(t)$, in thousands	Projected Population based on Exponential Function, $g(t)$, in thousands	Census Population Data and Intercensal Estimates for California (thousands)
1935 $t=35$	$103.2 \cdot 35 + 1490 = 5102$	$1490 \cdot 1.044^{35} = 6,725$	6,175
1960 $t=60$	$1032 \cdot 60 + 1490 = 7682$	$1490 \cdot 1.044^{60} = 19,734$	15,717
2010 $t=110$	$103.2 \cdot 110 + 1490 = 12,842$	$1490 \cdot 1.044^{110} = 169,919$	37,253

d. Which is a better model to for the population growth of California in 1935 and in 1960?

(handwritten) EXPONENTIAL

e. Does either model closely predict the population in 2010? Look at the graph on the right. What happened?

(handwritten) No. Growth slowed down...

California Population Growth: 1900-2010

(graph: Population in Thousands (y-axis, 5,000 to 40,000) vs Year (x-axis, 1880 to 2020))

Closure:

What is the difference between the way a linear function increases and the way an exponential function increases? Does an exponential function always grow faster than a linear function?