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POLYNOMIALS AND POLYNOMIAL OPERATIONS STUDY GUIDE

POLYNOMIALS

Polynomials are classified by two different categories: by the number of terms, and the degree of the leading exponent.

Number of Terms	Classification	Example
1	monomial	$4x^5$
2	binomial	$x^2 - 4$
3	trinomial	$x^2 - 3x + 2$

Degree of a Polynomial: the greatest sum of the exponents on the variables in each term.
Examples: $5x^4 + 3x^3$ is a fourth degree binomial.
$3x^3y^2 + 5y^4 - 3y^3$ is a fifth degree trinomial.

ADDING AND SUBTRACTING POLYNOMIALS

Add and Subtract Polynomials by doing the following:

- Combine like terms

Example: $4x^2 - 3x^3 + 2x^2 - 4x + 3 - 6x = \mathbf{-3x^3 + 6x^2 - 10x + 3}$

- Distribute any subtraction signs

Example: $(4x^2 - 3x + 2) - (2x^2 - 3x + 3) = \mathbf{4x^2 - 3x + 2 - 2x^2 + 3x - 3}$

- Rewrite in standard form

Example: $3x - 3x^2 + 17 - 2x^4 = \mathbf{-2x^4 - 3x^2 + 3x + 17}$

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MULTIPLYING POLYNOMIALS

Use either distribution or an array model.

Use **distribution**	$(y-5)(y+3)$ $= y(y+3) - 5(y+3)$ $= y^2 + 3y - 5y - 15$ $= y^2 - 2y - 15$
Use an **array model**	$(y-5)(y+3)$

	y	**−5**
y	y^2	$-5y$
3	$3y$	-15

SOLVING FOR VARIABLES

Use properties of equality and properties of arithmetic to solve for variables.

**If necessary, factor out a variable.
Example: Solve for *x*:

$$3x + xy = 21y - z$$

$$x(3+y) = 21y - z$$

$$x = \frac{21y - z}{3 + y}$$

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PASCAL'S TRIANGLE/BINOMIAL THEOREM

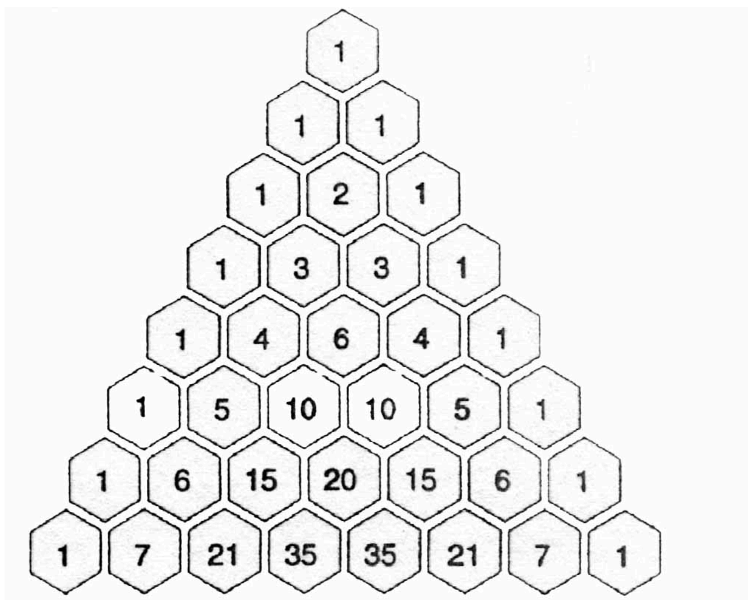
Pascal's triangle can help you determine the coefficients of variable terms in a binomial expansion.

Example:

$(x + y)^4$

$= (x + y)(x + y)(x + y)(x + y)$

$= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$



A few other things about Pascal's Triangle:

- The number of terms in your expansion will be one more than the exponent (i.e. if you are multiplying a binomial by itself 14 times, there will be 15 terms in your expansion.

- The exponent in the leading term of the binomial will decrease by one in each term, and the exponent in the second term will increase by one in each term.

Example:

$(x + 3)^5$

$= 1x^5 3^0 + 5x^4 3^1 + 10x^3 3^2 + 10x^2 3^3 + 5x^1 3^4 + 1x^0 3^5$

…and then simplify as needed.

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PROBLEM SET

Finish all problems by April 17. Let me know if you have any questions or difficulties in solving these problems. Please read all questions and follow instructions. Thanks!

1. Consider the term:

$$(2x-3y)^4$$

a. How many terms will there be in the expansion?

5 terms.

b. Expand using Pascal's Triangle, show your steps (Refer to the example on page 3 if needed).

$$(2x)^4+4(2x)^3(-3y)+6(2x)^2(-3y)^2+4(2x)(-3y)^3+(-3y)^4$$

$$16x^4+4(8x^3)(-3y)+6(4x^2)(9y^2)+4(2x)(-27y^3)+81y^4$$

$$\mathbf{16x^4-96x^3y+216x^2y^2-216xy^3+81y^4}$$

Since we were raising the binomial to the 4th power, I knew that there would be 5 terms, and that the coefficients would be 1, 4, 6, 4, and 1 before we account for the coefficients of the binomial. Then I looked at the leading term in the binomial ($2x$) and raised it to the 4th power, and then decreased the power by one in each successive term. I did the opposite to the second term of the binomial ($3y$), beginning with the 0 power, and increasing the power by one in each successive term.

c. Expand using distribution **by first obtaining two identical trinomials.**

$$(2x-3y)(2x-3y)(2x-3y)(2x-3y)$$

$$=(4x^2-12xy+9y^2)(4x^2-12xy+9y^2)$$

$$=(4x^2\cdot 4x^2)+\left(4x^2\cdot(-12xy)\right)+(4x^2\cdot 9y^2)+\left((-12xy)\cdot 4x^2\right)$$
$$+\left((-12xy)\cdot(-12xy)\right)+\left((-12xy)\cdot 9y^2\right)+(9y^2\cdot 4x^2)$$
$$+\left(9y^2\cdot(-12xy)\right)+(9y^2\cdot 9y^2)$$

$$=\mathbf{16x^4-96x^3y+216x^2y^2-216xy^3+81y^4}$$

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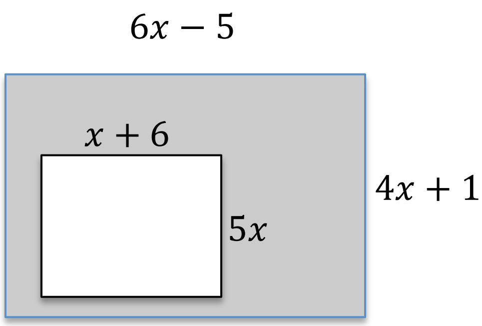
2. Find the variable expression that would best represent the area of the shaded region below.



Find the area of the smaller rectangle and then subtract that amount from the area of the larger rectangle.
Smaller rectangle: $5x(x+6) = 5x^2 + 30x$

Larger rectangle: $(6x-5)(4x+1) = 24x^2 - 20x + 6x - 5 = 24x^2 - 14x - 5$

Shaded area: $(24x^2 - 14x - 5) - (5x^2 + 30x)$

$$= \mathbf{19x^2 - 44x - 5 \text{ square units}}$$

3. Rewrite each of the expressions below by factoring out a common term. An example is shown below.
Example: $x^3 - 2x^2 + 12x = x(x^2 - 2x + 12)$

Answers can vary!! Answer shown is the most factored way.
a. $12x^2 - 20x^3$

$$\mathbf{4x^2(3 - 5x)}$$

b. $3x(12y - 5z) + 3x(2a + 7b)$

$$\mathbf{3x[(12y - 5z) + (2a + 7b)]}$$

c. $12(35x + 3y) + 24(3x - 2y)$

$$\mathbf{12[(35x + 3y) + 2(3x - 2y)]}$$
$$\mathbf{= 12[(35x + 3y) + (6x - 4y)]}$$
$$\mathbf{= 12(41x - y)}$$

d. $(2z - 5)(w + 3) - (3w + 11)(2z - 5)$

$$\mathbf{(2z - 5)[(w + 3) - (3w + 11)]}$$
$$\mathbf{= (2z - 5)(-2w - 8)}$$

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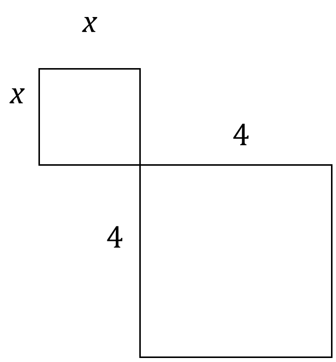
e. $(3x-5)(3y+2)+(6x-10)(5y+12)$

$$\mathbf{(3x-5)[(3y+2)+2(5y+12)]}$$
$$\mathbf{=(3x-5)[(3y+2)+(10y+24)]}$$
$$\mathbf{=(3x-5)(13y+26)}$$

4. Consider the following array model:

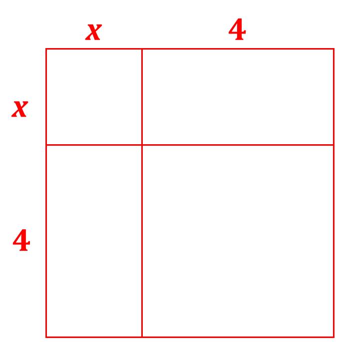


a. What is the correct **binomial expression** that represents this diagram?

$$\mathbf{x^2+16}$$

b. Hunter thought the above diagram represented the expression: $(x+4)^2$. Use distribution AND an array model to demonstrate what he did incorrectly.

i. diagram:



ii. Distribution:

$$\mathbf{(x+4)(x+4)}$$
$$\mathbf{=(x\cdot x)+(x\cdot 4)+(4\cdot x)+(4\cdot 4)}$$
$$\mathbf{=x^2+8x+16}$$

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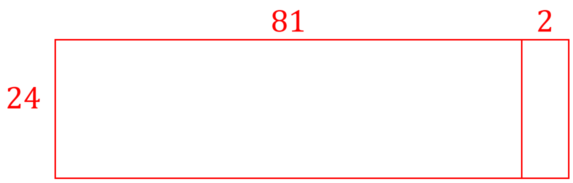
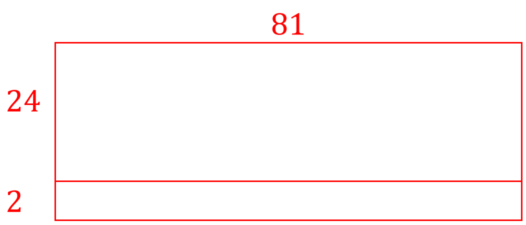
5. Which amount is greater: 24 × 83 or 26×81? Do NOT use the multiplication to answer this question!! **Draw two diagrams to explain your answer**.

24 × 83 is equivalent to (24×81) + (24×2)



26 × 81 is equivalent to (24 × 81) + (2×81)



It's clear to see that 26×81 is larger… 2×81 is larger than 2×24

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6. Answer parts (a)-(f).

a. Fill in the blanks: 8,943 = 8 ×1000 + 9 ×100 + 4 ×10 + 3 ×1.

b. Fill in the blanks: $7,427 =$ 7 $\times 10^3 +$ 4 $\times 10^2 +$ 2 $\times 10^1 +$ 7 $\times 10^0$.

c. What happens when we multiply each number above by 10? Explain using words. In the process of your explanation, please use the words "place value" and "base-10".

The place value moves to the left by one. Multiplying 8,943 by 10 equals 89,430. Because numbers use base 10, multiplying by 10 will simply increase the place value by one by adding a zero at the end.

d. Multiply the following: $x(8x^3 + 9x^2 + 4x + 3)$.

$$8x^4 + 9x^3 + 4x^2 + 3x$$

e. What is the **base** of the polynomial? How is this similar/different to your answer in part (c)?

It looks like the base of the polynomial is base *x*. It is similar to part (a) above and looks like what happens when you multiply part (a) by 10. And since the numbers behaved the way they did in base 10 when we multiplied by 10, it looks like we're dealing with base *x* because the polynomial is behaving similarly to the number.

f. Referring the part (d) above, what is the value of the expression if $x = 10$?

$$8x^4 + 9x^3 + 4x^2 + 3x$$

$$= 8(10)^4 + 9(10)^3 + 4(10)^2 + 3(10)$$

$$= 8(10{,}000) + 9(1{,}000) + 4(100) + 3(10)$$

$$= 80{,}000 + 9{,}000 + 400 + 30$$

$$= 89{,}430$$